Peristaltic Transport of a Rheological Fluid: Model for Movement of Food Bolus Through Esophagus

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Abstract

Fluid mechanical peristaltic transport through esophagus has been of concern in the paper. A mathematical model has been developed with an aim to study the peristaltic transport of a rheological fluid for arbitrary wave shapes and tube lengths. The Ostwald-de Waele power law of viscous fluid is considered here to depict the non-Newtonian behaviour of the fluid. The model is formulated and analyzed with the specific aim of exploring some important information concerning the movement of food bolus through the esophagus. The analysis has been carried out by using lubrication theory. The study is particularly suitable for cases where the Reynolds number is small. The esophagus is treated as a circular tube through which the transport of food bolus takes places by periodic contraction of the esophageal wall. Variation of different variables concerned with the transport phenomena such as pressure, flow velocity, particle trajectory and reflux are investigated for a single wave as well as for a train of periodic peristaltic waves. Locally variable pressure is seen to be highly sensitive to the flow index 'n'. The study clearly shows that continuous fluid transport for Newtonian/rheological fluids by wave train propagation is much more effective than widely spaced single wave propagation in the case of peristaltic movement of food bolus in the esophagus.

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1 Introduction

Swallowing of food is a mechanical process that begins with chewing, smashing and mixing of food in the oral cavity. Complex structural motion is set in within the pharynx that forces the food bolus rapidly into the esophagus. The process ends with the movement of the bolus into the stomach by peristaltic contraction of the esophageal wall. Pumping through various vessels of the physiological system by means of propagation of peristaltic waves is considered by physiologists as a natural mechanism of pumping materials in the case of most fluids of the physiological system. Besides physiological applications, the benefit of studies on peristaltic movement, however, extends to a variety of industrial appliances, e.g. roller pumps used to pump caustic or corroding liquids. Many of the essential fluid mechanical characteristics of peristalsis have found important applications in different engineering problems investigated by several researchers. Studies on peristalsis have also many important applications in the design and construction of many useful devices of biomedical engineering and technology, such as artificial blood devices, for example, finger pumps used in the pumping of blood. Our earlier communications (Misra et al. [1, 2, 3], Maiti and Misra [4]) and also those of some other authors [5, 6, 7, 8] provide useful information regarding peristaltic transport of various types of fluids.

Nomenclature	
R, θ, Z	Cylindrical co-ordinates
a	Average radius of the food bolus
H	Displacement of the esophageal wall in the radial direction
n	Fluid index number
k	Reciprocal of n
P	Fluid pressure
Q_1	Volume flow rate
t	Time
V_B	Volume of fluid within a single peristaltic wave (the bolus)
U, V, W	Velocity components in Z-, R-, θ- directions respectively
δ	wave number
ΔP	Pressure difference between the ends of the esophagus
ϵ	Minimum vessel radius (during occlusion)
λ	Wave length of the travelling wave motion in the esophagus
μ	Viscosity of the fluid (food bolus)
ν	Kinematic viscosity of the fluid (food bolus)
ϕ Wave amplitude	
ρ	Fluid density

Solid/liquid food mixture or chyme transport through esophagus which is a muscular conduit leading to the stomach takes place by means of progression of peristaltic contraction waves of circular muscle fibers contracted within circular muscle layers of the esophageal wall. When peristaltic waves start propagating, the circular muscle cells shorten themselves causing contractile forces. Involvement of both the nerve control and the intrinsic properties of muscle cells complicates the mechanism of muscle contraction. Consequently the peristaltic contraction acts as an external force on the tissue structure and travels downwards with a certain speed. The length of the esophagus is 250-300 mm for an adult human being. When stretched, it becomes more or less a straight tube that extends between the pharynx and the stomach. The two ends of the esophagus are controlled by the upper esophageal sphincters (UES) and the lower esophageal sphincters (LES). During resting condition, a high contractile pressure of at least 30 mm Hg is maintained. The intraluminal pressure at rest above the UES is maintained equal to the atmospheric pressure. In the thorax the luminal pressure at rest is typically slightly below

the atmospheric pressure, while in the abdomen the pressure is about 10 mm Hg above the atmospheric pressure. The thoracic as well as the intra-abdominal pressures are adjusted by respiration and is maintained at about 5 mm Hg. During the pharyngeal phase of swallowing, a mass of food that has been chewed at the point of swallowing, called bolus passes rapidly through the pharynx. Thereby the UES relaxes to atmospheric pressure, and the bolus arrives at the esophagus. As the intra-bolus pressure adds to about 5 mm Hg, a peristaltic contraction wave passes through the UES and then progresses down the esophagus at a rate of 20-40 mm/s, transporting the fluid bolus distally. Following the initiation of swallowing, the LES actively relaxes in a while to gastric pressure and discloses as the esophageal peristaltic wave forces the bolus into the stomach [9]. Esophageal peristalsis acts as a pump in transporting a fluid bolus from the upper esophagus to the stomach. Total pumping is not achieved, if the esophagus fails to maintain complete occlusion. Often in the region where the aortic arch impresses upon the esophagus, the esophageal wall occlusion remains incomplete. As a result, some fluid bolus leaks proximally through the contracted region and is left behind. When LES that helps keep the acidic contents of the stomach out of the throat does not work properly, laryngopharyngeal reflux occurs.

This leads to various discomforts of the body. For example, an individual may feel bitter test in the throat, uneasiness in swallowing of food bolus, feel burning sensation/pain in the throat and other similar health problems related to stomach. It may be mentioned that most of the studies on peristaltic transport made by previous authors are not very suitable for applications to those physiological situations where a single wave travels down the length of an organ having finite dimensions (e.g. esophagus). Li and Brasseur [10] dwelt on the said aspects of peristaltic pumping. They presented a model of peristaltic transport of a Newtonian viscous fluid, for arbitrary wave shapes/arbitrary wave number through a finite length tube. The conventional sinusoidal wave equation was developed by considering the position of the wall as a function of the minimum radius of the tube, which vibrates only in one direction. This study has got limited application. It is applicable only when the intake is water or some drink having similar physical properties.

But the movement of food grain bolus, like whipped cream, custard, ketchup, suspensions of corn starch and various masticated food-grains through the esophageal tube exhibits non-Newtonian behaviour. It is, therefore, important to study the peristaltic transport of the food bolus, when the motion is predominantly non-Newtonian. While studying the rheological behaviour of some physiological fluids, Patel et al. [11] carried out experimental investigation and

reported some data for some biorheological fluids. These data indicated that the masticated food-grains may be treated as a power-law fluid, where the power-law index may vary, depending on the type of the food material.

Keeping this in view, a mathematical model has been developed here to study the peristaltic transport of food bolus through the esophagus, by considering the motion to be governed by Ostwald-de Waele power law [12]. The fluid transport by peristalsis has been approximated by the lubrication theory which holds for Re$\leq$ 1 (cf. [13]). It may be mentioned that that when the average bolus is almost equal to the wave length ($\delta \sim 1$), the lubrication theory gives a reasonably good approximation to the pressure field (cf. [14]). The present study is undertaken to address the basic fluid mechanical issue of the non-steady effects corresponding to the finite tube length in the case of rheological (non-Newtonian) fluids. Our prime concern has been to examine the difference of the magnitudes of the flow variables in the cases of Newtonian and rheological (non-Newtonian) fluids. Particular emphasis has been paid to investigate the variation of essential local variable pressure together with volume flow rate, the pressure difference between the ends of the tube, representing the esophagus, the velocity distribution, the particle trajectories and the reflux phenomenon. Based upon the present study, a useful comparison has been made between the single wave and wave train effects on the peristaltic transport characteristics of the movement of food bolus.

2 Mathematical Modelling

In studies pertaining to friction dominated flows where axial length scale of velocity variation is large in comparison to the radial scales, use of the lubrication theory has been found to be very effective. It is known that transport of food bolus through esophagus takes place by the mechanism of peristalsis, where viscosity crosses the threshold limit 200 cp. The peristaltic wave speed c (characteristic velocity) in this case is normally 20-40 mm/s so that Reynolds number is of order 0.001-1. Let us treat the esophagus as an axi-symmetric tube of length L (which usually ranges between 250 mm and 300 mm) and denote by ϵ the minimum tube radius (i.e tube occlusion) and the wave number by $\delta = a/\lambda$. The ratio between average bolus radius $a = (V_B/\pi\lambda)^{\frac{1}{2}}$ (5-10 mm) and the typical wave length λ (50-100 mm) is of order 0.05-0.2, where V_B stands for the fluid volume within a single peristaltic wave (bolus).

We take (R,θ,Z) as the cylindrical coordinates of some location of a fluid particle, R being the radius of the tube; Z is measured in the direction of wave propagation. The schematic diagram

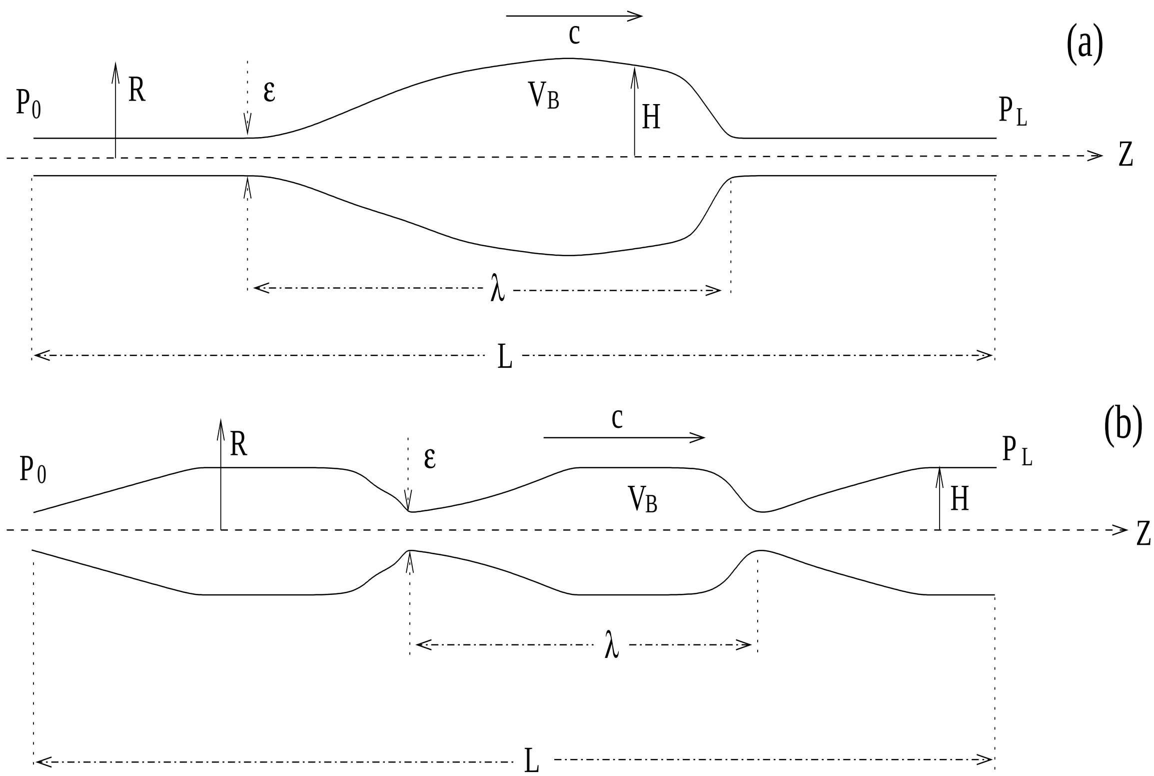


Figure 1: Schematic diagram of the problem: (a) a single contraction wave, (b) wave train. In both the cases, the food bolus is supposed to move from left to right against a pressure difference $P_L - P_0$, by peristaltic contraction waves along the tube. A non-integral number of peristaltic waves in the tube (L/λ) is depicted in (b).

of peristaltic transport has been given in Fig. 1 that illustrates (a) a single wave moving along a finite tube and (b) continual production of multiple waves. Let R=H(Z,t) denote the shape of the esophageal wall.

The mathematical model developed here pertains to a situation, where the fluid mechanical peristaltic transport of a food bolus is driven by arbitrarily shaped deformation of the wall of the esophagus. The pressure boundary conditions at the ends of the esophagus will also be considered, when its length is taken to be finite. The food bolus will be treated as an incompressible viscous Ostwald-de Waele type of rheological fluid [12]. If τ be the stress tensor and Δ the symmetric rate of deformation tensor, the constitutive equation for the fluid can be written as

$$\tau = \alpha \left\{ |\sqrt{\frac{1}{2}(\Delta : \Delta)}|^{n-1} \right\} \Delta, \tag{1}$$

$$where \ \ \frac{1}{2}(\Delta : \Delta) = 2\left(\left(\frac{\partial V}{\partial R}\right)^2 + \left(\frac{V}{R}\right)^2 + \left(\frac{\partial U}{\partial Z}\right)^2\right) + \left(\frac{\partial U}{\partial R} + \frac{\partial V}{\partial R}\right)^2$$

in which α and n denote respectively the consistency factor and the power law index parameter, depicting the behaviour of the fluid. It is known that a shear thinning fluid is characterized by $n < 1$, while for a shear thickening fluid, $n > 1$. Based on the above consideration, the motion of food bolus in the esophagus can be considered to be governed by the equations

$$\rho\left(\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial Z} + V\frac{\partial U}{\partial R}\right) = -\frac{\partial P}{\partial Z} + \frac{1}{R}\frac{\partial (R\tau_{RZ})}{\partial R} + \frac{\partial \tau_{ZZ}}{\partial Z} \tag{2}$$

$$\rho\left(\frac{\partial V}{\partial t} + U\frac{\partial V}{\partial Z} + V\frac{\partial V}{\partial R}\right) = -\frac{\partial P}{\partial R} + \frac{1}{R}\frac{\partial (R\tau_{RR})}{\partial R} + \frac{\partial \tau_{RZ}}{\partial Z} \tag{3}$$

3 Analysis

In the model, each material point on the wall of the esophagus is considered to move in the radial direction with velocity $\partial H(Z,t)/\partial t$. The following non-dimensional variables will be introduced in the analysis that follows:

$$\begin{aligned} \bar{Z} &= \frac{Z}{\lambda}, \ \ \bar{R} = \frac{R}{a}, \ \ \bar{U} = \frac{U}{c}, \ \ \bar{V} = \frac{V}{c\delta}, \ \ \delta = \frac{a}{\lambda}, \ \ \bar{P} = \frac{a^{n+1}P}{\alpha c^n \lambda}, \bar{Q} = \frac{\eta Q_1}{\pi a^2 c} \\ \bar{t} &= \frac{ct}{\lambda}, \ \ \bar{H} = \frac{H}{a}, \ \ \ Re = \frac{\rho a^n}{\alpha c^{n-2}}, \bar{\tau_0} = \frac{\tau_0}{\alpha(\frac{c}{a})^n}, \tau_{\bar{R}\bar{Z}} = \frac{\tau_{RZ}}{\alpha(\frac{c}{a})^n}, \end{aligned} \tag{4}$$

where $\eta = 1$ for wave train and $\eta = L/\lambda$ for single wave movement. In terms of these variables the governing equations can be rewritten as (dropping the bars over the symbols)

$$Re\delta\left(\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial Z}+V\frac{\partial U}{\partial R}\right)=-\frac{\partial P}{\partial Z}+\frac{1}{R}\frac{\partial\left(\Phi\left(R\frac{\partial U}{\partial R}+R\delta^2\frac{\partial V}{\partial Z}\right)\right)}{\partial R}+2\delta^2\frac{\partial\left(\Phi\frac{\partial U}{\partial Z}\right)}{\partial Z} \tag{5}$$

$$Re\delta^3\left(\frac{\partial V}{\partial t}+U\frac{\partial V}{\partial Z}+V\frac{\partial V}{\partial R}\right)=-\frac{\partial P}{\partial R}+\delta^2\frac{1}{R}\frac{\partial(R\Phi\frac{\partial V}{\partial R})}{\partial R}+\delta^2\frac{\partial\left(\Phi(\frac{\partial U}{\partial R}+\delta^2\frac{\partial V}{\partial Z})\right)}{\partial Z} \tag{6}$$

$$\Phi=\left|\sqrt{2\delta^2\left\{\left(\frac{\partial V}{\partial R}\right)^2+\left(\frac{V}{R}\right)^2+\left(\frac{\partial U}{\partial Z}\right)^2\right\}+\left(\frac{\partial U}{\partial R}+\delta^2\frac{\partial V}{\partial Z}\right)^2}\right|^{n-1} \tag{7}$$

Considering the wall curvature as very small ($\delta \ll 1$), it is possible to apply the lubrication theory, where the inertial effect is negligible and the dominant radial scale 'a' is quite small, in comparison to the dominant axial scale λ. In such a case, the distribution of pressure is uniform on each cross section. Under these considerations, the governing equations and the boundary conditions in terms of non-dimensional variables reduce to the following set of equations :

$$0=-\frac{\partial P}{\partial Z}+\frac{1}{R}\frac{\partial(R\frac{\partial U}{\partial R}|\frac{\partial U}{\partial R}|^{n-1})}{\partial R} \tag{8}$$

$$0=-\frac{\partial P}{\partial R} \tag{9}$$

$$\frac{\partial U}{\partial R}=0,\ V=0\ at\ R=0;\quad U=0\ ,\ V=\frac{\partial H}{\partial t}at\ R=H \tag{10}$$

$$P=P_0\ at\ Z=0\ and\ P=P_L\ at\ Z=L \tag{11}$$

By solving (8) subject to the conditions (10) and (11), we find the velocity field in the form

$$U(R,Z,t)=\frac{p|p|^{k-1}}{2^k(k+1)}\left[R^{k+1}-H^{k+1}\right] \tag{12}$$

$$V(R,Z,t)=\frac{Rp|p|^{k-1}}{2^k(k+1)}\left[p_1\left(\frac{H^{k+1}}{2}-\frac{R^{k+1}}{k+3}\right)+\frac{k+1}{2}H^k\frac{\partial H}{\partial Z}\right] \tag{13}$$

where $p=\frac{\partial P}{\partial Z}$, $k=\frac{1}{n}$ and $p_1=k\frac{\partial p}{\partial Z}/p$.

Now using the last of the conditions (10), we have from (13) the equation

$$\frac{\partial H}{\partial t}=\frac{H^{k+1}p|p|^{k-1}}{2^{k+1}(k+3)}\left[p_1H+(k+3)\frac{\partial H}{\partial Z}\right] \tag{14}$$

The pressure gradient p obtained on integrating (14) is given by

$$p|p|^{k-1} = \frac{2^{k+1}(k+3)}{H^{k+3}}[c_1 + \int_0^Z H\frac{\partial H}{\partial t}dZ], \tag{15}$$

where c_1 is, in general, a function of time t. Solving (15), we obtain

$$P(Z,t) - P(0,t) = \int_0^Z p(S,t)dS$$
$$= \int_0^Z \left[\left| \frac{2^{k+1}(k+3)}{H^{k+3}} \left\{ c_1 + \int_0^S H\frac{\partial H}{\partial t}dZ \right\} \right|^{n-1} \left\{ \frac{2^{k+1}(k+3)}{H^{k+3}} \left\{ c_1 + \int_0^S H\frac{\partial H}{\partial t}dZ \right\} \right\} \right] dS \tag{16}$$

Using (12) along with (15), the non-dimensionalized volume flow rate is given by

$$\bar{Q}(Z,t) = 2\eta \int_0^H RUdR$$
$$= -\frac{\eta p|p|^{k-1}H^{k+3}}{2^k(k+3)} \tag{17}$$
$$= -2\eta \left\{ c_1 + \int_0^Z H\frac{\partial H}{\partial t}dZ \right\} \tag{18}$$

Putting Z=0, the instantaneous flow rate at the inlet of the esophagus is given by

$$\bar{Q}(0,t) = -2\eta c_1 \tag{19}$$

In terms of the flow rate $\bar{Q}$(0,t) at the inlet, the temporal flow rate $\bar{Q}(Z,t)$ at any position of the esophagus can be expressed as

$$\bar{Q}(Z,t) = \bar{Q}(0,t) - 2\eta \int_0^Z H\frac{\partial H}{\partial t}dZ \tag{20}$$

Using (16), (19) and (20), the flow rate $\bar{Q}(Z,t)$ is found to be related to the pressure P(Z,t) as

$$P(Z,t) - P(0,t) = -\int_0^Z \left| \frac{2^k(k+3)\bar{Q}(Z,t)}{\eta H^{k+3}} \right|^{(1/k)-1} \left\{ \frac{2^k(k+3)\bar{Q}(Z,t)}{\eta H^{k+3}} \right\} dZ \tag{21}$$

Thus the pressure difference between the esophageal ends is given by

$$\Delta P = P(L,t) - P(0,t)$$
$$= -\int_0^L \left| \frac{2^k(k+3)\bar{Q}(Z,t)}{\eta H^{k+3}} \right|^{(1/k)-1} \left\{ \frac{2^k(k+3)\bar{Q}(Z,t)}{\eta H^{k+3}} \right\} dZ \tag{22}$$

It is worthwhile to note that the equation (17) reduces to the corresponding equation derived by Li and Brasseur [10] who studied a similar problem for a Newtonian fluid.

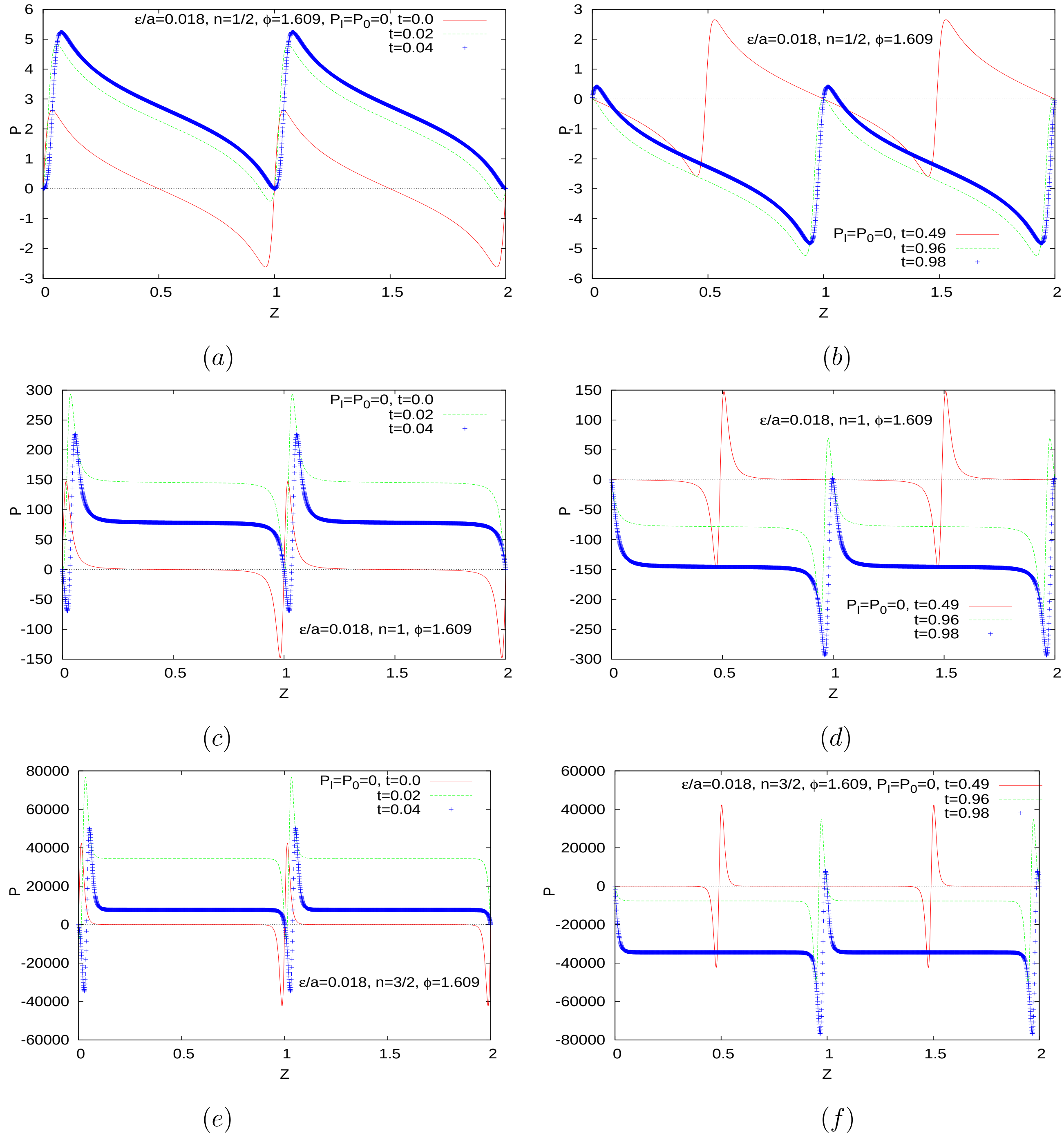


Figure 2: Local Pressure Distribution in the case of a wave train in the esophagus. These figures reveal that for a shear thinning fluid the (global) maximum and minimum peaks are attained respectively at t=0.4 and t=0.96. This is contrary to the cases of Newtonian and shear thickening fluids for which the maximum and minimum values are attained at t=0.02 and t=0.98 respectively.

4 Numerical Study

This section is devoted to a quantitative analysis of the mathematical model presented in the earlier sections. We shall try to investigate the difference between the characteristics in the cases of single wave and multiple wave (i.e. wave train) for the peristaltic transport of food bolus for arbitrary wave shapes and esophageal length. It may be noted that unlike in the study of Li and Brasseur [10] (for the Newtonian case), it is not possible to find a closed form solution for c_1 that appears in (15), (16) and (18). Consequently it is not possible to find an explicit analytical expression for the fluid flux $\bar{Q}$ when the tube length and wave shape are both arbitrary. Determination of the quantitative estimates of different physical variables has been based upon the consideration that for the rheological (non-Newtonian) fluid taken up in our present study the flow rate $\bar{Q}(Z,t)$ is given by

$$\bar{Q}^n(Z,t) = Q^n + H^2 - \frac{1}{\eta}\int_0^{\eta} H^2 dt, \tag{23}$$

Q being the time-averaged volume flow and the superscript 'n' denoting the power law index of the fluid.

4.1 Pressure Distribution

Let us first investigate the effect of finite tube length on the pressure distribution during the peristaltic transport. It may be noted that pressure is essentially a mechanical variable in the functioning of the esophagus where intraluminal manometry is used as a common diagnostic tool in order to obtain the contractile characteristics of the circular muscle within the esophageal wall. Let us first take up the case of an integral number of train waves moving with constant speed through a tube having finite length whose ends are subjected to constant pressure of equal magnitude. During peristalsis, the esophagus is considered to be of sinusoidal shape defined by the equation

$$H(Z,t) = \epsilon/a + 0.5\phi\{1 - \cos 2\pi(Z-t)\} \tag{24}$$

In order to keep the fluid volume fixed within one wave period, ϕ is adjusted when ϵ/a changes. When two waves are present in the esophagus, Figs. 2(c-d) give the pressure variation of the fluid (considered Newtonian) at six specially chosen locations during one wave period. The graphs indicate that within one wave period, there are two peaks in the pressure distribution within one wave period with a gradual pressure ramp in between and transition occurs from

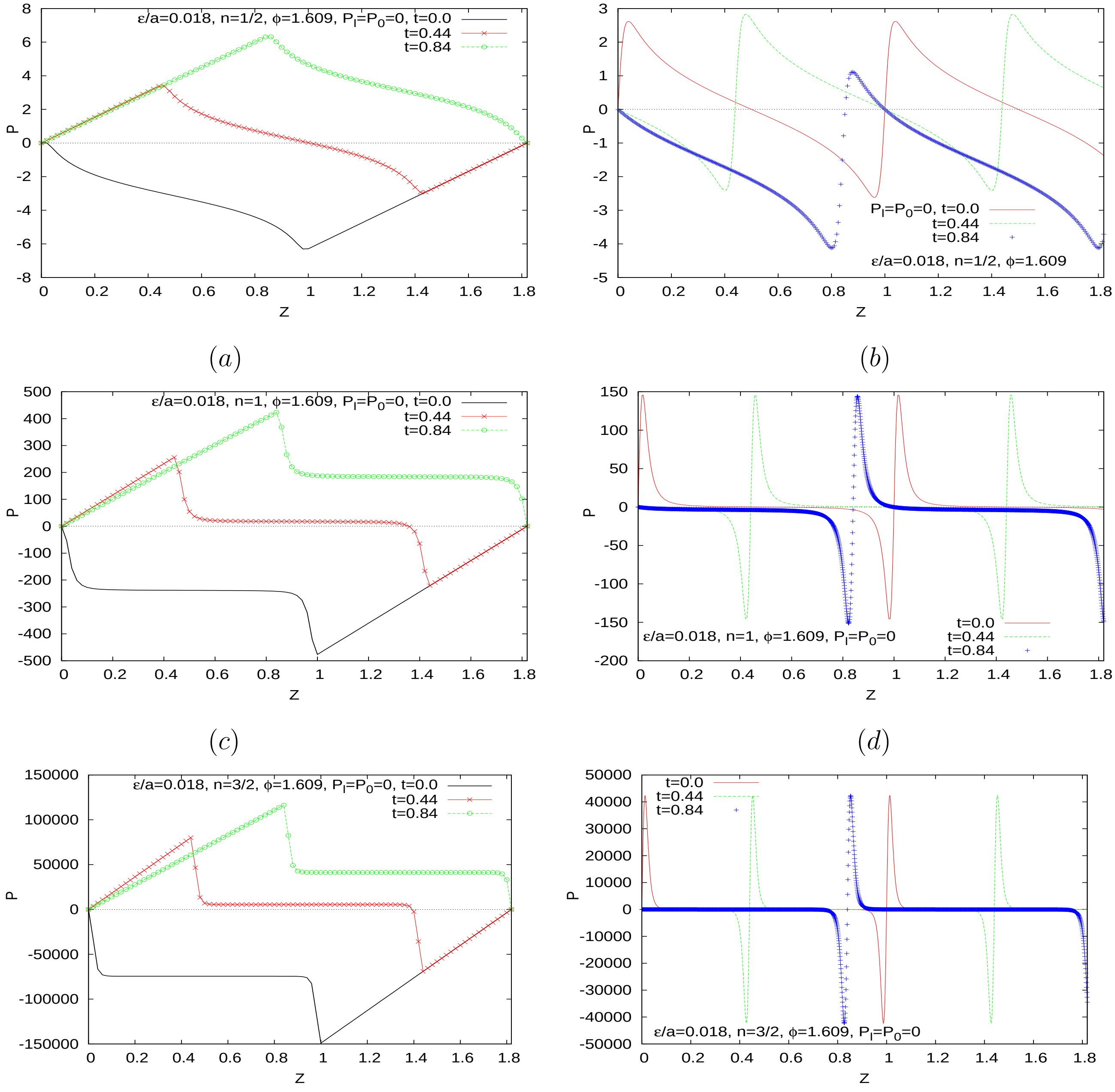


Figure 3: Local pressure distribution in the esophagus in the case of a single wave. These findings match quite fairly with those of a Newtonian fluid reported earlier by Li and Brasseur [10]. For a shear-thinning fluid (cf. (a-b):n=1/2) and a shear-thickening fluid (cf. (e-f): n=3/2) the overall behavior is found to be somewhat similar. Moreover, for both the cases of single wave and wave train propagation, while for a shear-thinning fluid (n=1/2), the magnitude of pressure is quite small (nearly 1.2-2.5% of a Newtonian fluid), for a shear-thickening fluid (n=3/2), it is quite large (nearly 260-300 times that for the Newtonian case).

large minimum peak to large maximum peak at the point of maximum occlusion within the contraction zone. It is further seen that to the right of this point the wall of esophagus moves radially inward ($\partial H/\partial t < 0$) presumably owing to the contraction of the circular muscle. As a result of this, a large pressure gradient is created there. To the left of the point of minimum radius the wall moves radially outwards. It causes a corresponding drop in pressure there. Therefore local instantaneous motion occurs to the left of the point of maximum occlusion and also to the right at the remaining portion of the region. The net averaged flow over one wave period takes place towards the wave. We find that the results obtained on the basis of the present analysis and the form of $\bar{Q}(Z,t)$ given by (23) match with those reported in [13]. A comparison of variation of pressure between Newtonian and rheological (non-Newtonian) fluids suggest that pressure is highly sensitive to the rheological fluid index 'n'. Although the nature of pressure change along the tube length is almost similar, it is noted from Figs. 2(a-b) that the amount of change is very small for a shear-thinning fluid with n=1/2. The change has been observed clearly throughout the tube. For a shear-thickening liquid with n=3/2, Figs. 2(e-f) show that the magnitude of pressure is very large compared to that for a Newtonian fluid.

In order to discuss the significant differences in reflux and and pumping phenomena between single and wave train peristaltic transport of rheological fluids, it is worthwhile to compare both the spatial and temporal pressure variations. Figs. 3 present the comparison, where the spatial variations in pressure are given at fixed times for single bolus transport and train wave transport with a non-integral number of waves in the esophagus. For the purpose of comparison of the results of the present study for the rheological fluid ((non-Newtonian) with those for the Newtonian fluid, the results obtained in [10] are reproduced on the basis of our present study for the Newtonian case in Figs. 3(c-d). An extended adverse pressure gradient for a Newtonian fluid (cf. Fig. 3(c)) is found to be created by the peristaltic wave from the inlet of the esophagus to its tail and the outlet of the esophagus to the head of the peristaltic wave. Thereby the motion is opposite to that of the peristaltic wave in the said region. Fig. 3(d) shows that in the wave train case, these extended regions are absent and during its transit to the outlet of the esophagus a single peristaltic wave is followed by an ever-increasing region of backward motion. Some portion of the backward motion remains at the outlet until the peristaltic wave head reaches the outlet. In the contrary, when the bolus passes through the outlet of the esophagus, it carries the fluid along with it that leads the net transport in the direction of the wave.

It is noticed that unlike single wave propagation, wave train has as many pairs of peaks of pressure in both Newtonian and rheological fluids as the number of waves present in the duct

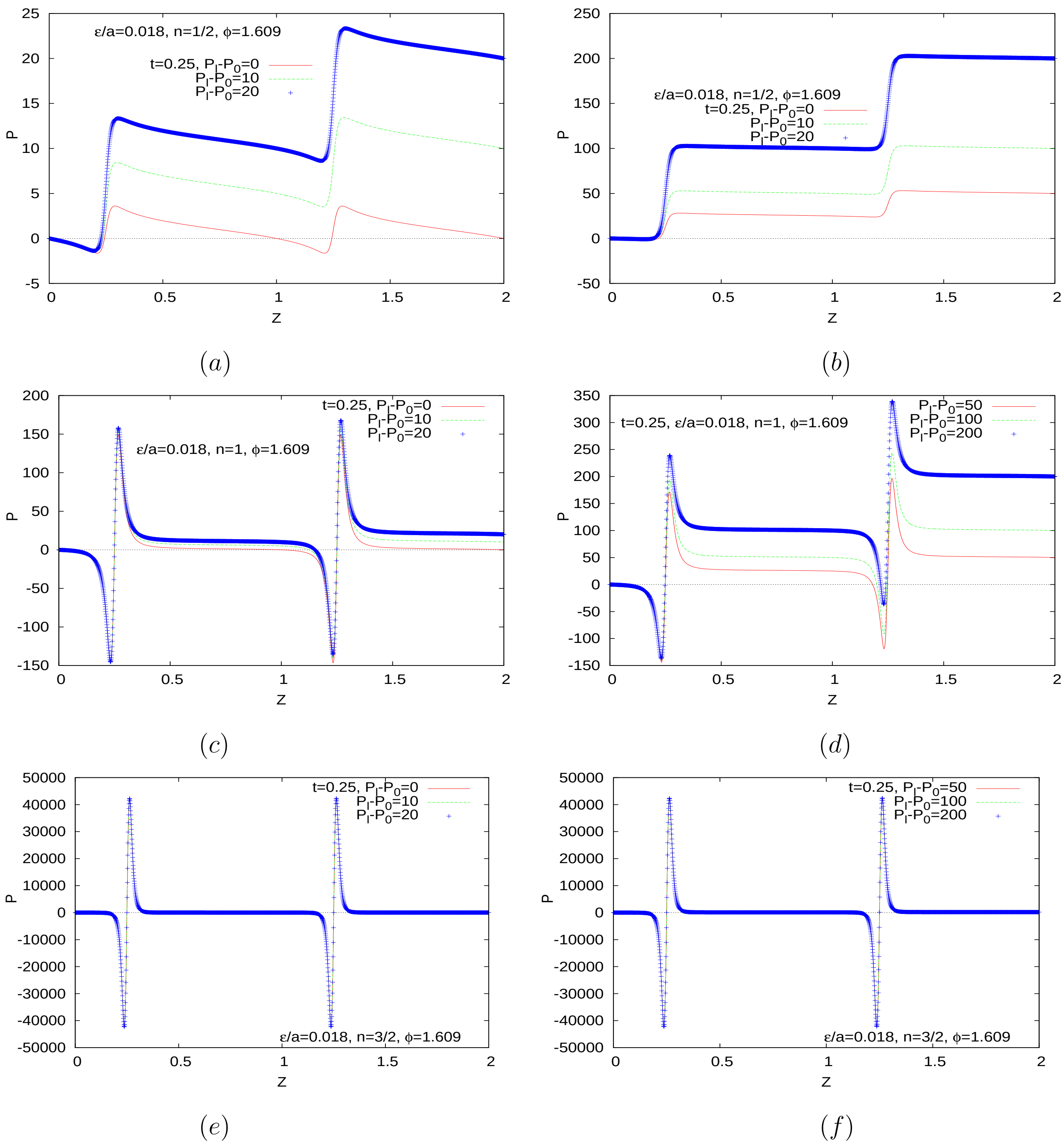


Figure 4: Local pressure distribution in the case of a wave train propagation during movement of the food bolus. Pressure rise at the lower end of the esophagus (LEE) leads to an enhancement of pressure throughout the length of the esophagus.

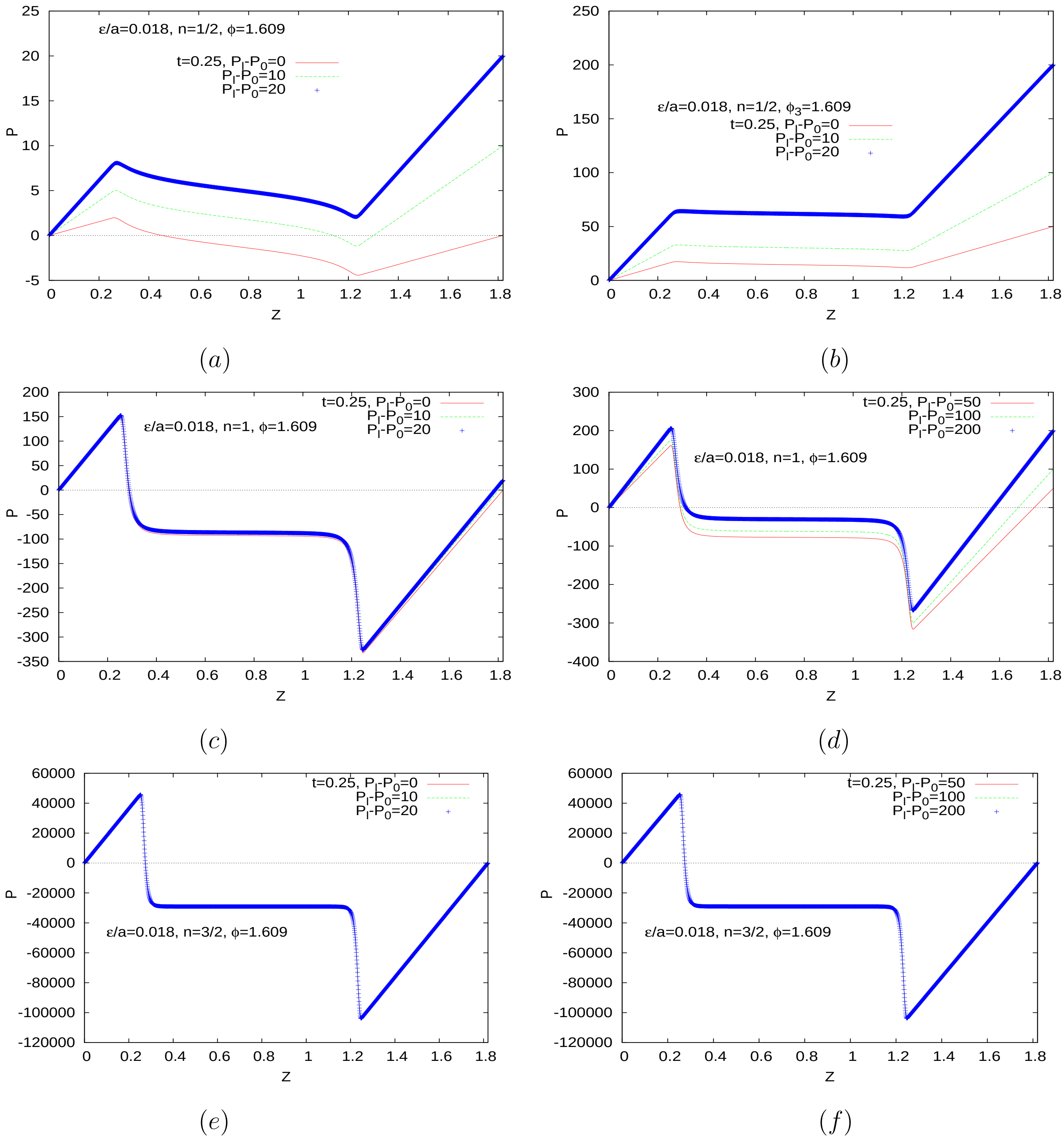


Figure 5: Local pressure distribution in the case of a single wave during food bolus movement. As pressure at the lower end of esophagus (LEE) rises, region of adverse pressure gradient is extended rapidly for a shear thinning fluid, while for a Newtonian fluid it is significant only after the pressure at LEE (P_l) exceeds a critical value.

in a given time interval. Moreover, in case of train wave propagation, the pressure transit at once from minimum to maximum when the wave head is immediately succeeded by the tail of the leading wave for all types of fluids examined here.

A negative pressure difference drives a positive flow, whereas a positive pressure difference creates the resistance of the flow. When it attains a certain critical value (that depends on the wave amplitude), the power-law index and other related conditions, there is a possibility that the flow would be completely restrained. If it exceeds that critical value, the flow will take place in the backward direction. This causes emesis (in clinical terms), which is commonly known as vomiting that involves forceful expulsion of the contents of the stomach through the esophagus. Physiologically it occurs due to gastritis, or poisoning, or due to high intracranial pressure or over exposure to conizing radiation. This may be also happen to patients suffering from brain tumor. The backward flow of undigested food from the stomach to the mouth is, however, called medically as regurgitation. Figs. 4 and Figs. 5 exhibit local pressure distribution when the pressure at the lower end is greater than that at upper end of the esophagus for wave train transport and for single bolus transport respectively. For a shear thinning fluid Figs. 4(a-b), it is seen that local pressure enhances significantly with the increase in ΔP. For a Newtonian fluid (cf. Figs. 4(c-d)), it is also increases with the increase in ΔP except at the transition region. However, in the case of shear thickening fluid (n=3/2), Figs. 4(e-f) indicate that value of ΔP considered here (i.e. $0 \leq \Delta P < 200$) does not significantly affect the pressure. In the case of a single bolus transport, it is noted from Figs. 5 that as the pressure at the lower end of esophagus (LEE) increases, local pressure throughout the region also increases for shear thinning fluid when n=1/2, where as for Newtonian fluid this increase is significant when P_1 reaches a greater value.

4.2 Velocity Distribution

Since the velocity profiles, the pressure and the esophageal radius, all change with time, it is pertinent to investigate the distribution of velocity at different time intervals of a wave period. Moreover, for a single wave transport, the limited region where peristaltic wave is active deserve special attention. In the case of free pumping ($\Delta P = 0$) for a single wave at t=0.0, Fig. 6(a) shows that in the region where the tube radius is minimum, the flow takes place in opposite direction but the magnitude of velocity is small, whereas in the remaining region the magnitude of the velocity is large and direction of velocity is the same as that of the propagating wave.

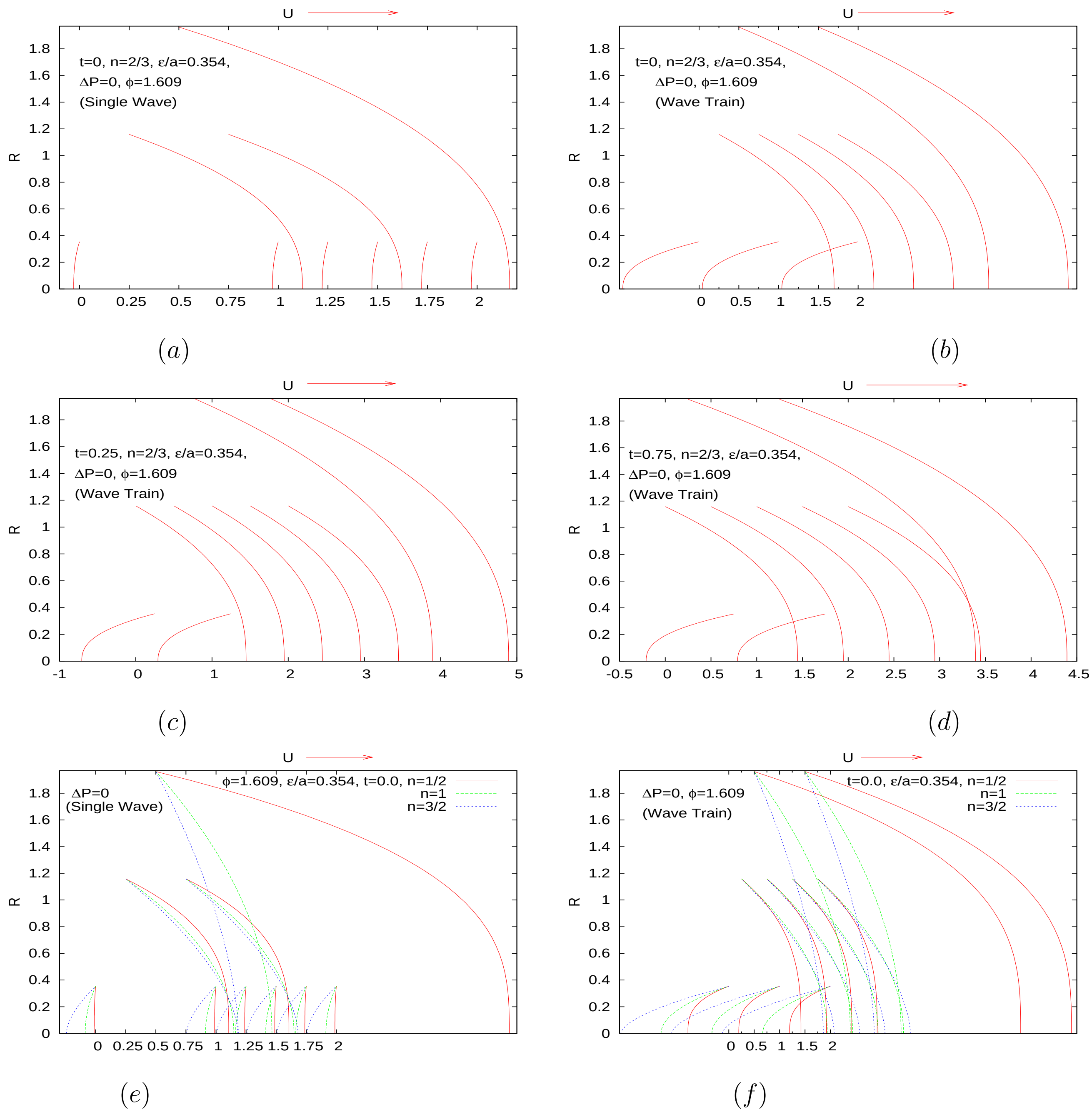


Figure 6: Distribution of axial velocity at different instants of time. Fig. (a) shows that in the region where the tube radius is minimum, the flow takes place in opposite direction but the magnitude of velocity is small, whereas in the remaining region the magnitude of the velocity is large and direction of velocity is the same as that of the propagating wave. Thus in the case of a single wave when the esophagus fails to maintain total occlusion, fluid transport takes place in the forward direction where the wave is active, while in other parts of the tube, the bolus moves slowly in the backward direction.

Thus in the case of a single wave when the esophagus fails to maintain total occlusion, fluid transport takes place in the forward direction where the wave is active, while in other parts of the tube, the bolus moves slowly in the backward direction. As time progresses, although this trend is maintained, the regions in which forward and backward flows occur, change depending on the current position of the single wave. In the contrary, for a wave train, the transport takes place with very high velocity in both the forward and backward regions (cf. Fig. 6(b)). It may be noted that backward flow occurs mainly between the junction of the wave lengths, although occurrence of forward and backward flow regions is similar to the single wave case. However, some difference in the regions of forward and backward velocity profiles of forward and backward motions is observed with the passage of time (cf. Figs 6(b-d)). Figs. 6(e-f) indicate that as the fluid index number 'n' increases, backward flow is enhanced, while the forward flow reduces significantly, whether it is a case of single wave propagation or that of a wave train propagation.

Figs. 7-8 present velocity distribution for situations where the pressure at the lower end of esophagus (LEE) is higher than that at the UEE (upper end of esophagus). When ΔP rises, reflux region is extended for single bolus transport as well as train wave transport for all types of fluids including shear thickening case (although there is not significant pressure change due to increase in P_1). In addition, magnitude of velocity is reduced in the forward flow region, while in the reflux region it increases.

4.3 Particle Trajectory and Reflux phenomenon

It is known that one of the important characteristics of peristaltic flow is the reflux phenomenon. It refers to the presence of fluid particles that move in a direction opposite to that of the peristaltic wave. In the infinite tube model reflux generally occurs under conditions of partial occlusion and adverse pressure difference across one wave length. The comparison of particle trajectories corresponding to three different types of fluids in esophagus are shown in Figs. 9-11 for a wave train propagating with sinusoidal shape, where non-integral number of waves exists in the tube. To determine the trajectories of the particles in the Lagrangian frame of reference, the simultaneous differential equations

$$\frac{dZ}{dt} = U, \quad \frac{dR}{dt} = V \tag{25}$$

have been solved by using RK4 method successively, starting from the initial location of the particles. The particles are initially taken to be located in the vicinity of the most occluded

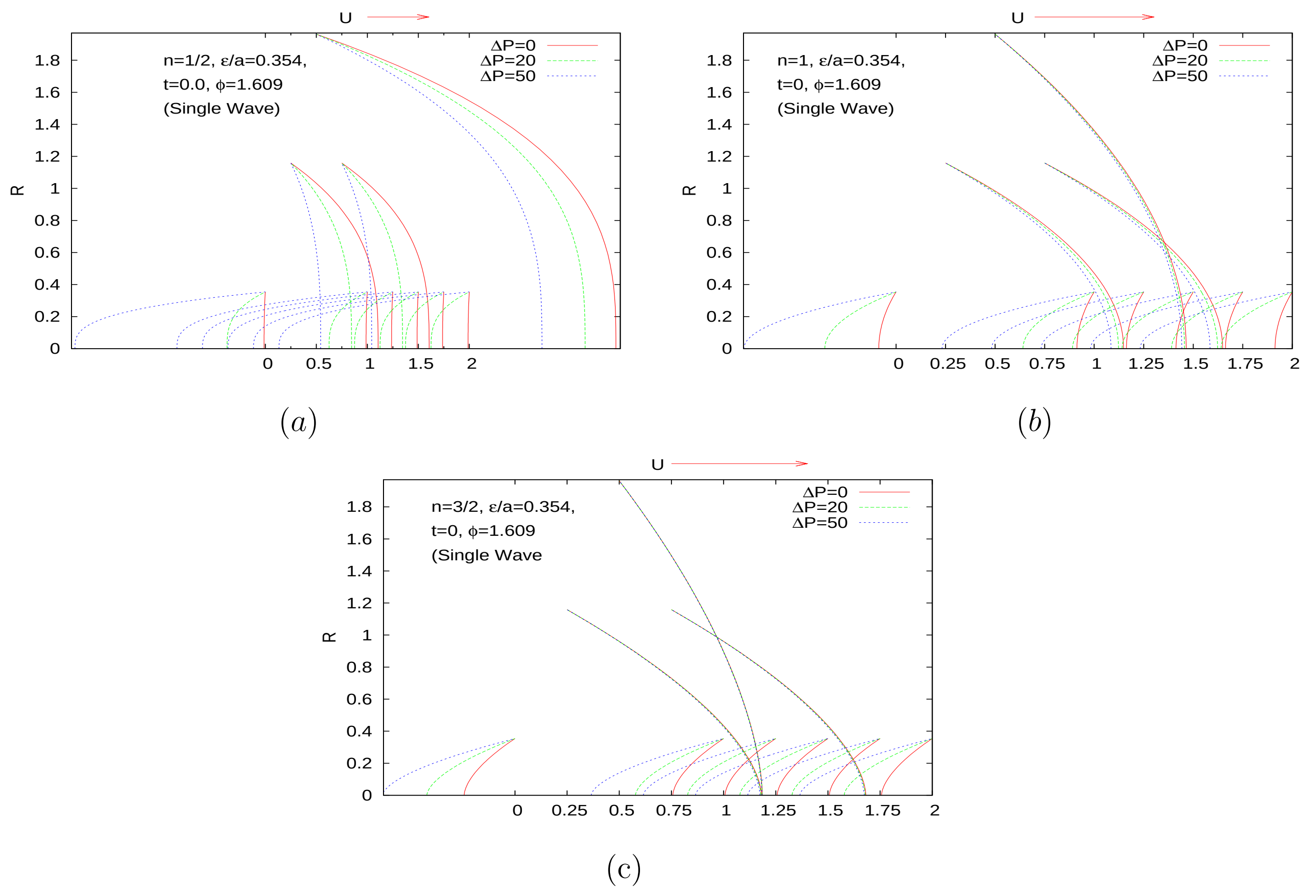
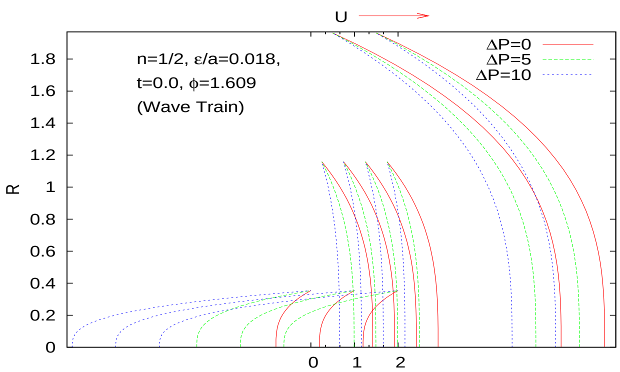
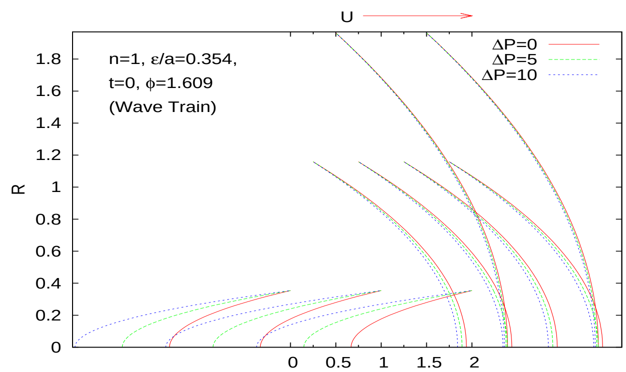


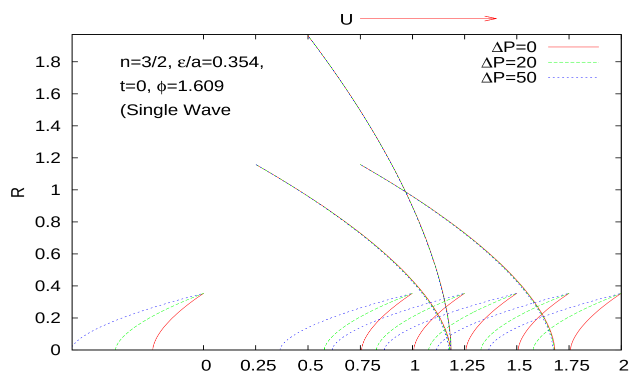
Figure 7: In the case of a large pressure gradient at the two ends of the tube, where the pressure at the lower end is higher, the reflux region is prominent in the case of a single bolus transport. This is in the contrast to the case when the pressure gradient is zero.



(a)



(b)



(c)

Figure 8: Distribution of axial velocity at different instants of time in the case of wave train propagation. When the pressure at the lower end of esophagus (LEE) is more than that at the upper end of esophagus (UEE), backward flow is enhanced, while the forward flow reduced. It is interesting to note that backward flow occurs at a faster rate even there is a small change of pressure at the LEE.

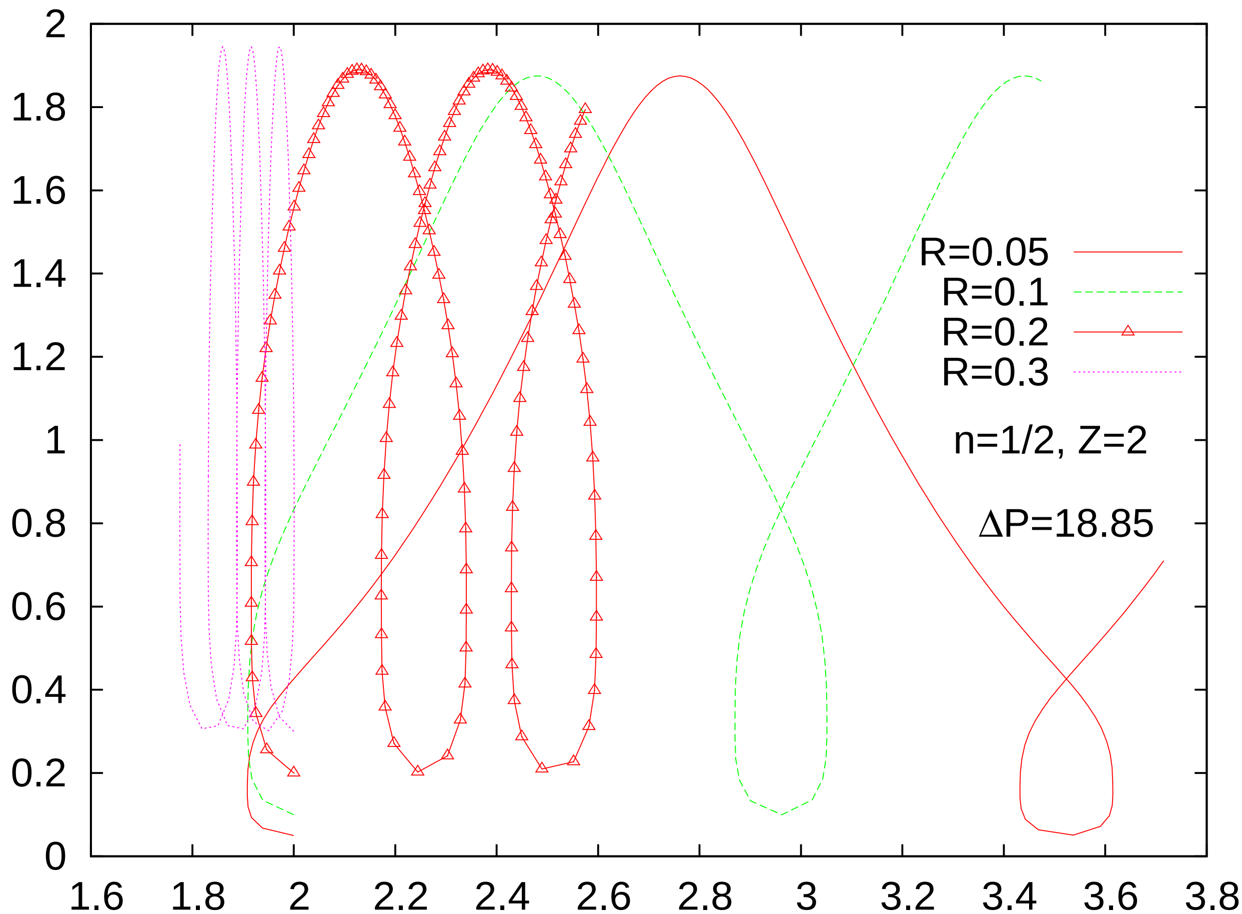


Figure 9: Particle trajectories for peristaltic flow of a non-Newtonian fluid of shear-thinning type at different locations (Z,R) (viz. (2,0.05), (2,0.1), (2,0.2), (2,0.3)). Particles near the tube-axis travel more distance in the axial direction and time taken by them to complete the respective trajectories is less than their own particle periods. Where as, particles near the boundary move less distance in the axial direction and time taken to complete the trajectories is more than the respective particle periods.

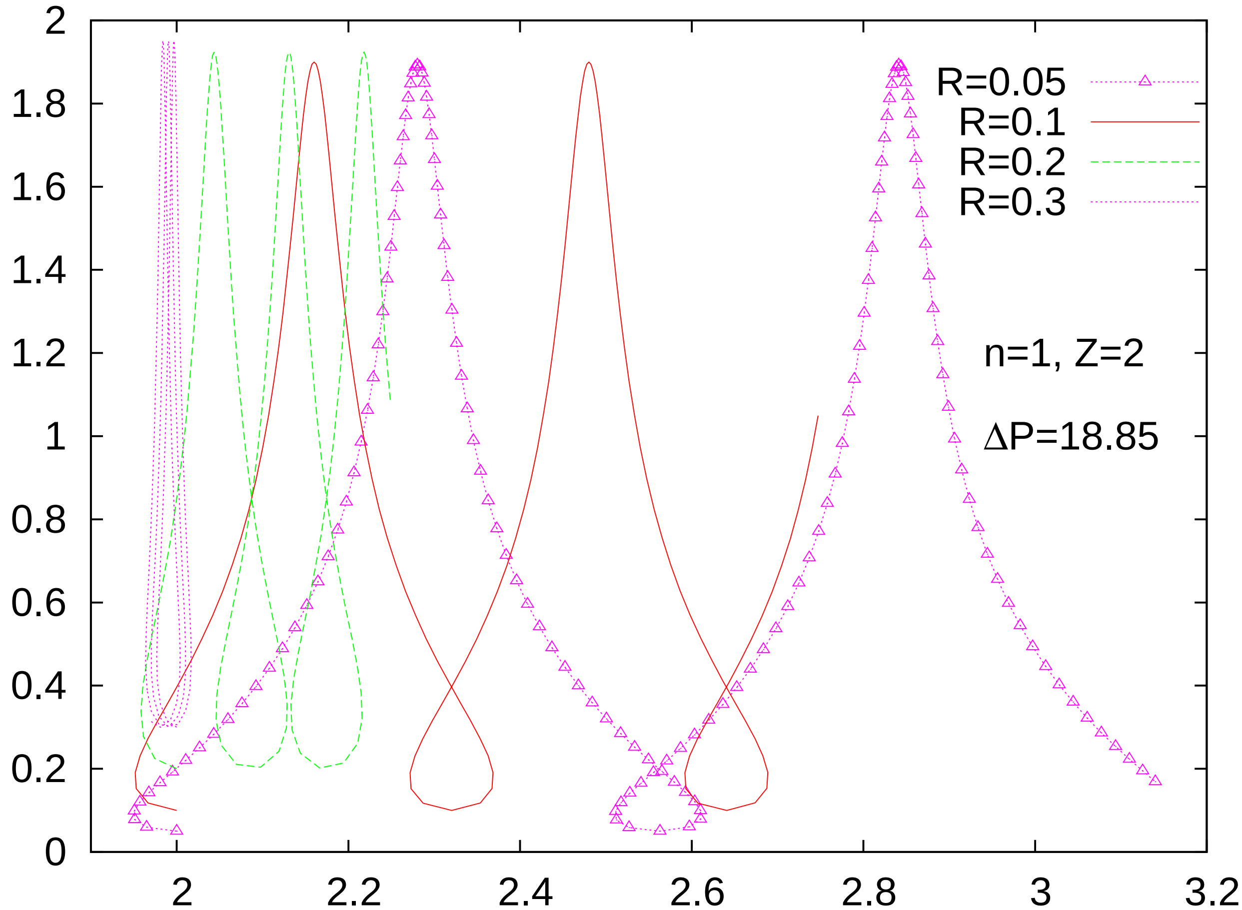


Figure 10: Particle trajectory for peristaltic flow of a Newtonian fluid at different locations. Final position of the particle at (2,0.3) (located near the boundary) is (1.97,0.43). This particle slightly moves in the direction opposite to the wave.

point. The results presented in Figs. 9-11 have been computed by taking the dimensionless pressure difference ΔP to be 18.85 (cf. [10]). It may be noted that the particle trajectories computed on the basis of the present study resemble those presented by Li and Brasseur [10]. It is also observed that most particles in both Newtonian and rheological fluids undergo a net positive displacement, while the particles nearest to the tube wall move in the direction opposite to that of wave propagation. Further, as rheological fluid index 'n' increases, axial displacement decreases and the particles reaching near the boundary start moving slowly towards the axis at some points of time. However, it is interesting to note that particles near the boundary for a shear-thickening fluid move in the forward direction. Beyond the most occluded region, it is observed that as the rheological fluid index 'n' increases, axial displacement of particles near the axis increases.

5 Concluding Remarks

The motivation behind the present investigation is to study the peristaltic transport of food bolus through the esophagus. For a non-Newtonian fluid, local pressure is found to be very much dependent on the fluid behaviour index 'n'. The study shows that variation in pressure (which is a local variable), forward and retrograde flows and particle trajectories of the food bolus are all highly sensitive to the length of the esophagus, the existence of integral and non-integral number of waves in the tube as well as the propagation of single/multiple waves in the esophagus. The leakage of fluid is of common occurrence in the neighbourhood of an aortic arch. The present study suggests that while designing a peristaltic pump for all types of Newtonian/rheological fluids, it is quite important to duly account for the unsteady effects arising out of the variation in the length of the finite esophageal tube as well as the differences between single and multiple peristaltic wave propagation.

The present study is important so for as the movement of food material through the esophagus is concerned, since due to shear thinning effect, the viscosity of the fluid decreases with increase in rate of shear stress. Tomato sauce/paste sauce, wheaped cream are some of the food materials that exhibit shear thinning effect. Corn starch has been used as a non-Newtonian fluid in some experimental studies. When water is mixed with cornstarch in a certain proportion, the mixture, termed as Oobleck, possesses the property of shear thickening.

When peristaltic waves start propagating, the circular muscle cells shorten themselves and generate the contraction force. Involvement of both the nerve control and intrinsic properties of

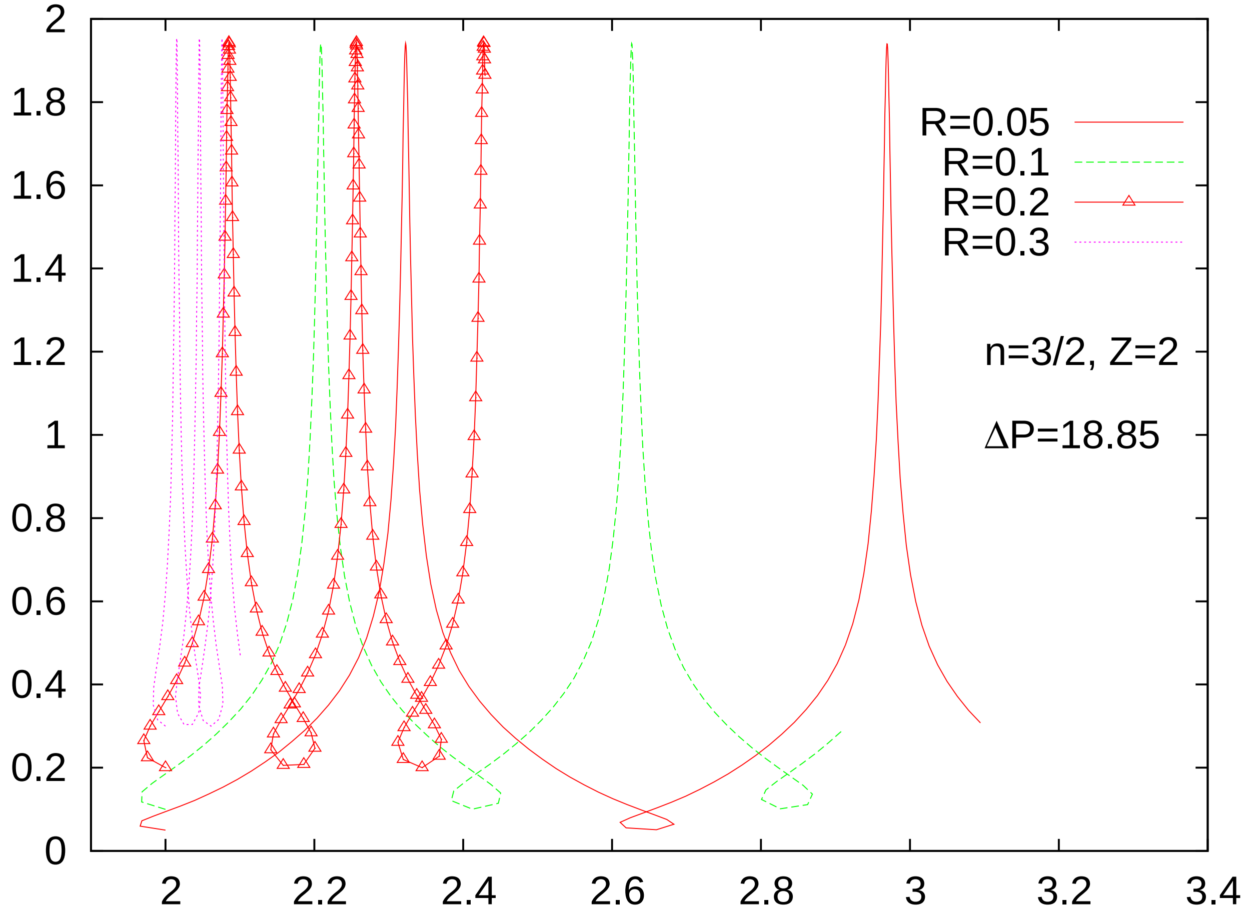


Figure 11: Particle trajectories for peristaltic flow of a shear-thickening fluid at different locations. Final position of the particle at (2,0.3) (located near the boundary) is (2.1,0.47). All particles shown here move in direction same as that of the wave propagation.

muscle cells makes the mechanism of muscle contraction somewhat complicated. The peristaltic contraction is believed to act as an external force on the tissue structure and travels downward with a certain speed. Peristalsis in esophagus normally occurs by the propagation of a single wave of active muscle contraction preceded by a single wave of muscle relaxation. The motion of the wall is directly linked to the pressures within the fluid (cf. [9]) and there is a relationship between the deformation of the esophageal wall as recorded radiographically and the intra-bolus pressures as measured manometrically during food bolus transport. Since the esophageal wall is actively forced and the cavity volume at the contraction region is forced to be reduced quickly in the contraction zone, occlusion pressures are high. The bolus geometries in the contraction region are associated with a rapid increase in pressure toward the point of maximum occlusion. On the contrary, relaxation is linked with a lack of muscle tone. The rate of local pressure is so less when the fluid is shear thinning ($n < 1$) and is very large when the fluid is shear thickening ($n > 1$) compared to a Newtonian fluid. Hence the muscle contracts at a very slower rate when $n < 1$, while it contracts rapidly when $n > 1$. Thus it is more comfortable to swallow a food material having shear thinning properties than food stuff possessing shear thickening characteristics. In the case of a Newtonian fluid, the comfort in swallowing is not as easy as in the case of shear thinning material, but is easier than in case of shear thickening food material. Thus the study of the present non-Newtonian model contributes to having a better understanding of muscle movement. Moreover, the esophageal wall just distal to the peristaltic wave must be passively forced open by the pressures within the approaching food bolus. In order to overcome the thoracic pressure exterior to the esophagus as well as any residual tension within the esophageal wall, these intra-bolus pressure needs to be sufficiently high. Figs. 2-5 reveal that at first the pressure rises slowly and then increases rapidly to a peak as the contraction wave passes, while relaxation is not related with rapid changes in pressure.

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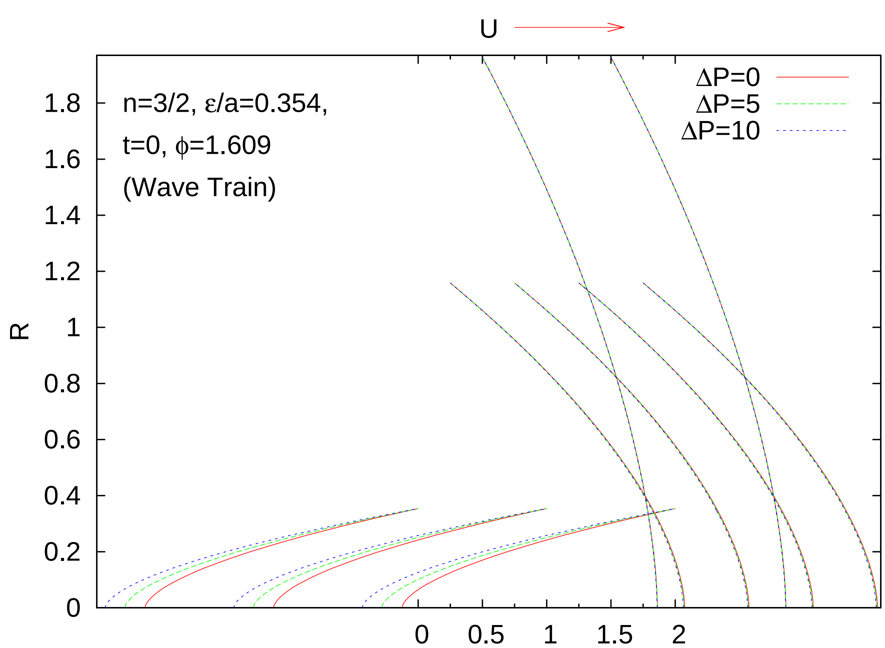
U
n=3/2, ε/a=0.354,
t=0, ϕ=1.609
(Wave Train)
ΔP=0
ΔP=5
ΔP=10
R
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
0.5
1
1.5
2